Healthcare and Consumption with Aging

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This is research on DEATH

- How to make life more pleasant?
  - consumption: feels good at the moment.
  - healthcare: defers death.
Do We Know Healthcare?

Journal of American Medical Association (JAMA)

- On July 11, 2016, Barack Obama published
  “United States Health Care Reform
  –Progress to Date and Next Steps”

- Editorial summary:
  4 big surprises from ACA
  - e.g. cost of healthcare ↓ while quality ↑.

All pundits of healthcare were WRONG about ACA
**Mortality v.s. Age**

- Exponential increase in age [Gompertz’ law]:

\[ dM_t = \beta M_t dt \quad (\beta \approx 7.1\%) \]
LITERATURE

▶ How Exogenous Mortality affects Consumption?
  ▶ Healthcare?

▶ Health as Capital, Healthcare as Investment
  ▶ Grossman (1972), Ehrlich and Chuma (1990)
  ▶ Health Capital observable?

▶ Mortality rates decline with health capital.
  ▶ Gompertz’ law?
**This Paper Idea**

- Household maximizes utility from lifetime consumption:

  \[ \sup_{c,h} \mathbb{E} \left[ \int_0^\tau e^{-\delta t} U(c_t X_t) dt \right]. \]

- Money can buy...
  - consumption, which generates utility...
  - healthcare, which reduces mortality growth...
    \[ \Rightarrow \text{buying time for more consumption.} \]

**Questions**

- Find optimal control processes

  \[ \{\hat{c}_t\}_{t \geq 0}, \{\hat{h}_t\}_{t \geq 0}. \]

- \( \{\hat{h}_t\}_{t \geq 0} \Rightarrow \text{endogenous mortality curve} \Rightarrow \text{follows Gompertz’ law?} \]
**The Value Function**

- Naïve approach:
  \[
  \sup_{c,h} \mathbb{E} \left[ \int_0^\tau e^{-\delta t} U(c_tX_t)dt \right].
  \]

- **NOT** invariant to utility translation!

  If \( U \) becomes \( U + k \),
  \[
  \sup_{c,h} \left\{ \mathbb{E} \left[ \int_0^\tau e^{-\delta t} U(c_tX_t)dt \right] + k\mathbb{E}\left[ \frac{1 - e^{-\delta \tau}}{\delta} \right] \right\}
  \]

**OBSERVE:**

\( \tau \) is endogenous \( \implies \) NO translation invariance.
The Value Function

Our approach:

- After death, household carries on with the same optimization problem.

\[
0 \quad \tau_1 \quad \tau_2 \quad \tau_3 \quad \cdots \cdots
\]

- Death scales household wealth by factor \( \zeta \in [0, 1] \).
  (\( \zeta \): inheritance tax, annuity loss, foregone income...)

- The value function:

\[
V(x, m) = \sup_{c, h} \mathbb{E} \left[ \sum_{n=1}^{\infty} \int_{\tau_{n-1}}^{\tau_n} e^{-\delta t} U(\zeta^n X_t c_t) dt \right] \quad \text{with } \tau_0 := 0.
\]

(Translation Invariant)
ASSUMPTIONS

- Simplifications:

\[ 0 \quad \tau_1 \quad \tau_2 \quad \tau_3 \quad \cdots \cdots \]

\[ x \quad \zeta X_{\tau_1} \quad \zeta^2 X_{\tau_2} \quad \zeta^3 X_{\tau_3} \quad \cdots \cdots \]

\[ m \quad M_{\tau_1} \quad M_{\tau_2} \quad M_{\tau_3} \quad \cdots \cdots \]

- Surviving spouse in similar age group.
- Most weight carried by first two lifetimes.

- Isoelastic utility:

\[ U(x) = \frac{x^{1-\gamma}}{1-\gamma} \quad 0 < \gamma \neq 1 \]
Mortality Dynamics

- Without healthcare, mortality grows exponentially [Gompertz’ law]:
  \[ dM_t = \beta M_t dt. \]

- Healthcare slows down mortality growth
  \[ dM_t = (\beta - g(h_t))M_t dt \]

- \( h_t \): healthcare-wealth ratio
- \( g : \mathbb{R}_+ \mapsto \mathbb{R}_+ \) measures efficacy of healthcare
  - \( g(0) = 0 \), \( g \) is increasing and concave.
  - Example:
    \[ g(h) = \frac{a}{q} h^q \quad a > 0, q \in (0, 1) \]
ASSUMPTIONS

Efficacy depends on healthcare-wealth ratio.

- Means-tested subsidies;
- Chetty et al. (2016, JAMA): life expectancy is significantly correlated with health behaviors but not with access to medical care.

⇒ $h_t$ reflects time and lost-income costs.
Wealth Dynamics

Household wealth grows at a constant interest rate $r > 0$, minus consumption and health spending:

$$dX_t = (r - c_t - h_t)X_t dt.$$
A Stochastic Control Problem

▶ The value function:

\[ V(x, m) = \sup_{c, h} \mathbb{E} \left[ \sum_{n=1}^{\infty} \int_{\tau_{n-1}}^{\tau_n} e^{-\delta t} U(\zeta^n X_t c_t) dt \right] \]

▶ State processes:

\[ dX_t = (r - c_t - h_t) X_t dt \quad X_0 = x, \]
\[ dM_t = (\beta - g(h_t)) M_t dt, \quad M_0 = m. \]

▶ Distributions of death times:

\[ \mathbb{P}(\tau_n > t \mid \tau_{n-1} < t) = \exp \left\{ - \int_{\tau_{n-1}}^{t} M_s ds \right\}. \]
3. Verification argument

Construct a solution to PDE

PDE for $V(x, m)$ (HJB eqn.)

2. ???
(Perron’s method in our case)

1. Dynamic Programming + Stochastic Calculus
**Dynamic Programming Principle**

\[
V(x, m) = \sup_{c,h} \mathbb{E} \left[ \sum_{n=1}^{\infty} \int_{\tau_{n-1}}^{\tau_n} e^{-\delta t} U(\zeta^n X_t c_t) dt \right]
\]

\[0 \quad T \quad \rightarrow \quad \text{time}\]

- DPP states:

\[
V(x, m) = \sup_{c,h,\pi} \mathbb{E} \left[ \int_0^T e^{-\int_0^t (\delta + M_s) ds} [U(c_t X_t) + M_t V(\zeta X_t, M_t)] dt 
+ e^{-\int_0^T (\delta + M_s) ds} V(X_T, M_T) \right].
\]
**Itô’s Formula**

- By Itô’s formula in stochastic calculus,

\[
d \left( e^{- \int_0^t (\delta + M_s) \, ds} V(X_t, M_t) \right) = e^{- \int_0^t (\delta + M_s) \, ds} \left[ - (M_t + \delta) V(X_t, M_t) + V_x(X_t, M_t) dX_t 
+ V_m(X_t, M_t) dM_t + \frac{1}{2} V_{xx}(X_t, M_t) (dX_t)^2 \right].
\]
“DPP + Itô’s Formula” yields

\[
0 = \sup_{c,h} \mathbb{E} \left[ \int_0^T e^{-\int_0^t (\delta + M_s) \, ds} \left( U(c_t X_t) + M_t V(\zeta X_t, M_t) \\
- (\delta + M_t) V(X_t, M_t) + [r + \mu \pi - c_t - h_t] X_t V_x(X_t, M_t) \\
+ (\beta - g(h_t)) M_t V_m(X_t, M_t) + \frac{1}{2} \sigma^2 \pi^2 x^2 V_{xx}(X_t, M_t) \right) \, dt \right].
\]

A Big Guess: \( V(x, m) \) is a solution to the PDE

\[
0 = \sup_{c,h \geq 0} \left\{ U(cx) + m V(\zeta x, m) - (\delta + m) V(x, m) \\
+ [r + \mu \pi - c - h] x V_x(x, m) + (\beta - g(h)) m V_m(x, m) \\
+ \frac{1}{2} \sigma^2 \pi^2 x^2 V_{xx}(x, m) \right\}.
\]
HAMILTON-JACOBI-BELLMAN EQUATION

- The HJB equation for $V$:

$$\sup_{c \geq 0} \left\{ U(cx) - hxV_x(x, m) \right\}$$
$$+ \sup_{h \geq 0} \left\{ -mg(h)V_m(x, m) - hxV_x(x, m) \right\}$$
$$- \delta V(x, m) + rxV_x(x, m)$$
$$+ (V(\zeta x, m) - V(x, m))m + \beta mV_m(x, m) = 0. \quad (pde)$$

- Optimal strategies:

$$\hat{c} = \frac{V_x(x, m)^{-\frac{1}{\gamma}}}{x}, \quad \hat{h} = (g')^{-1} \left( \frac{-xV_x(x, m)}{mV_m(x, m)} \right)$$


**Reduction to ODE**

- Taking $V(x, m) = \frac{x^{1-\gamma}}{1-\gamma} u(m)^{-\gamma}$ gives

\[
\begin{align*}
    u(m)^2 - & \left( \frac{\delta + (1-\zeta^{1-\gamma})m}{\gamma} + \left( 1 - \frac{1}{\gamma} \right) r \right) u(m) \\
   & + mu'(m) \left( \sup_{h \geq 0} \left\{ g(h) - \frac{1-\gamma}{\gamma} \frac{u(m)}{mu'(m)} h \right\} - \beta \right) = 0. 
\end{align*}
\]

- Optimal strategies:

\[
\begin{align*}
    \hat{c} = \frac{V_x(x, m)^{-\frac{1}{\gamma}}}{x} = u(m), \\
    \hat{h} = (g')^{-1} \left( -\frac{xV_x(x, m)}{mV_m(x, m)} \right) = (g')^{-1} \left( \frac{1-\gamma}{\gamma} \frac{u(m)}{mu'(m)} \right).
\end{align*}
\]
THREE SETTINGS

To understand effects of aging and healthcare, consider

1. **Forever Young** [Neither Aging nor Healthcare]:
   \[ M_t \equiv m \geq 0. \]

2. **Gompertz's law** [Aging without Healthcare]:
   \[ dM_t = \beta M_t dt, \quad M_0 = m > 0. \]

3. **The general case** [Aging with Healthcare]:
   \[ dM_t = (\beta - g(h_t))M_t dt, \quad M_0 = m > 0. \]
Neither Aging nor Healthcare

- $M_t \equiv m > 0$ (forever young).
- (ode) reduces to

$$u(m)^2 - \left( \frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left( 1 - \frac{1}{\gamma} \right) r \right) u(m) = 0.$$

- Optimal strategy:

$$\hat{c} = u(m) = c_0(m) := \frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left( 1 - \frac{1}{\gamma} \right) r.$$
AGING WITHOUT HEALTHCARE

- $dM_t = \beta M_t dt$.
- This is non-standard, cf. Huang, Milevsky, & Salisbury (2012)).
- (ode) reduces to

$$u(m)^2 - \left(\frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left(1 - \frac{1}{\gamma}\right)r\right)u(m) - \beta mu'(m) = 0.$$  

- Optimal strategy:

$$\hat{c} = u(m)$$

$$= c_\beta(m) := \left(\int_0^\infty e^{-\frac{(1-\zeta^{1-\gamma})my}{\gamma}}(\beta y + 1)^{-\left(1 + \frac{\delta + (\gamma - 1)r}{\beta \gamma}\right)} dy\right)^{-1}$$

- Asymptotics for old age (large $m$):

$$c_\beta(m) = c_0(m) + \beta + O\left(\frac{1}{m}\right).$$
Mortality and aging have large impacts on \( \hat{c} \).

- green curve: \( c_0 \)
- orange curve: \( c_\beta \)
AGING WITH HEALTHCARE

- $dM_t = (\beta - g(h_t))M_t dt$.
- **Assumption**: healthcare...
  - slows aging,
  - does not stop aging.
- **Intuitive idea**: a solution $u^* \text{ to (ode)}$ should satisfy
  $$c_0 \leq u^* \leq c_\beta, \quad u^* \text{ is concave}.$$
- **Observe**: Under the condition
  $$g\left((g')^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right) < \beta,$$
  - $c_\beta$ is a supersolution to (ode),
  - $c_0$ is a subsolution to (ode).
CONSTRUCTION OF $u^*$

- **Perron’s method:**

  $$u^*(m) := \inf_{u \in S} u(m) \quad m > 0,$$

  where $S$ is the collection of $u : \mathbb{R}_+ \mapsto \mathbb{R}_+$ satisfying

  - $c_0 \leq u \leq c_\beta$,
  - $u$ is a *viscosity* supersolution to (ode).
  - $u$ is concave, increasing.

  \[\implies u^* \text{ is a viscosity solution to (ode)}\]

- **Regularity:**

  **viscosity solution property + concavity**

  \[\implies u^* \text{ is a classical solution to (ode)}\]
Verification

- $u^*(m)$ is a classical solution to (ode).
- $\frac{x^{1-\gamma}}{1-\gamma} (u^*(m))^{-\gamma}$ is a classical solution to (pde).
- Using verification argument,

$$V(x, m) = \frac{x^{1-\gamma}}{1-\gamma} (u^*(m))^{-\gamma}, \quad (x, m) \in \mathbb{R}_+^2.$$
Main Result

Suppose $0 < \gamma < 1$ and $\bar{c} := \frac{\delta}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r > 0$. If

$$g \left( \left( g' \right)^{-1} \left( \frac{1 - \gamma}{\gamma} \right) \right) < \beta,$$  

then

$$V(x, m) = \frac{x^{1-\gamma}}{1-\gamma} u^*(m)^{-\gamma} \quad \text{for all } (x, m) \in \mathbb{R}^2_+,$$

where $u^* : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is the unique nonnegative, strictly increasing solution to

$$u(m)^2 - \left( \frac{\delta + (1 - \zeta^{1-\gamma}) m}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r \right) u(m)$$

$$+ mu'(m) \left( \sup_{h \geq 0} \left\{ g(h) - \frac{1 - \gamma}{\gamma} \frac{u(m)}{mu'(m)} h \right\} - \beta \right) = 0.$$
Main Result (conti.)

Furthermore, \((\hat{c}, \hat{h})\) defined by

\[
\hat{c}_t := u^*(M_t), \quad \hat{h}_t := (g')^{-1} \left( \frac{1 - \gamma}{\gamma} \frac{u^*(M_t)}{M_t(u^*)'(M_t)} \right), \quad t \geq 0,
\]

are optimal strategies.
CALIBRATION

- Take efficacy function as $g(h) := a \frac{h^q}{q}$, with $a > 0$, $q \in (0, 1)$.
- Take $r = 1\%$, $\delta = 1\%$, $\gamma = 0.67$, $\zeta = 50\%$ from literature.
- Calibrate $\beta, m_0, a, q$ to mortality rate data:

\[ \beta = 7.7\%, \quad m_0 = 0.019\%, \quad q = 0.46, \quad a = 0.1. \]
LONGER LIVES

Model explains decline in mortality at old ages.
**Optimal Strategies**

- Healthcare negligible in youth.
- Increases faster than consumption (in log scale!)
Healthcare as Fraction of Spending

- Convex, then concave; rises quickly to contain mortality.
- Slows down when cost-benefit declines.
THANK YOU!!

Q & A

Preprint available @ ssrn.com/abstract=2808362

“Healthcare and Consumption with Aging”